

# Planetary boundary layers: our habitat and coupling parts of climate machine

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# PBLs in Earth system

Atmospheric planetary boundary layers (PBLs) are strongly turbulent layers immediately affected by dynamic, thermal and other interactions with Earth's surface

**PBSs subject to diurnal variations**

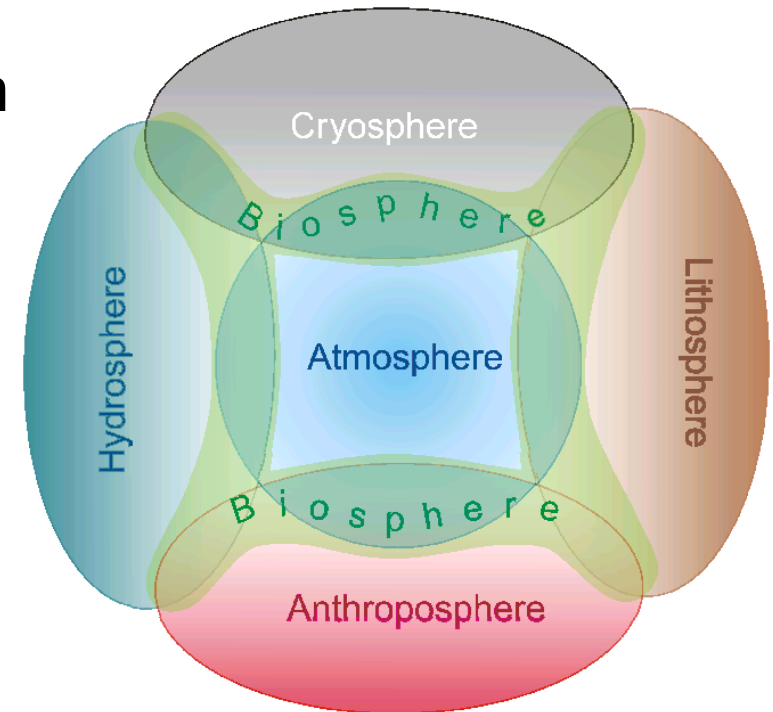
**Absorb surface emissions**

**Control microclimate, extreme colds and heats, air pollution**

**Are sensitive to human impacts**

**Have dozens to thousands *m* heights**

**Atmospheric and hydrospheric PBLs couple geospheres and include 90% biosphere and 100% anthroposphere**



# Content

**STABLE AND NEUTRAL stratification** – comparatively shallow PBLs affected by Earth's rotation and stratification

- Free-flow stratification
- Equilibrium PBL height regimes
- Non-steady regimes → prognostic PBL height equation

**UNSTABLE stratification** – deep, non-rotational, well-mixed, ever-growing convective layers

- PBL growth-rate

Buoyancy-budget models: prescribed entrainment

- Variable entrainment (PBL ventilation)

Buoyancy- and energy-budget models: modelled entrainment



# Different types of PBL

Self-organisation and non-local nature



Modern classification accounting for  $N$  distinguishes between nocturnal ↔ long-lived truly ↔ conventionally shear-free ↔ sheared

**NS**  $N = 0$  ↔ **LS**  $N > 0$  **TN**  $N = 0$  ↔ **CN**  $N > 0$

**cells** ↔ **rolls**

**STABLE**  $B_s < 0$

**NEUTRAL**  $B_s = 0$

**CONVECTIVE**  $B_s > 0$

Traditional classification of **PBLs** by sign of surface buoyancy flux  $B_s$  disregarded **free-flow Brunt-Väisälä frequency**  $N$  (at  $z > h$ )

Ekman (1905) set up a concept of planetary boundary layer (PBL) explain physics of Polar ice drift Earth's rotation



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# Different types of PBL

Classification by **sign of surface buoyancy flux**  $B_s$

**Stable**  $B_s < 0$

**Neutral**  $B_s = 0$

**Unstable (convective)**  $B_s > 0$

disregards **free-flow Brunt-Väisälä frequency**  $N$  (at  $z > h$ ).

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We account for  $N$  and distinguish

**Stable**

nocturnal stable (NS)  $N = 0$

long-lived stable (LS)  $N > 0$

**Neutral**

truly neutral (TN)  $N = 0$

conventionally neutral (CN)  $N > 0$

**Unstable** shear-free (convective cells) in two-layer fluid  $N = 0$

in stratified fluid  $N > 0$

**Unstable** sheared (convective rolls) in two-layer fluid  $N = 0$

in stratified fluid  $N > 0$

# Stable and neutral PBLs

Ekman (1905) rotational PBL height scale  $h_* = (K_* / |f|)^{1/2}$

$f = 2\Omega \sin\varphi$  is Coriolis parameter,  $K_*$  is eddy viscosity

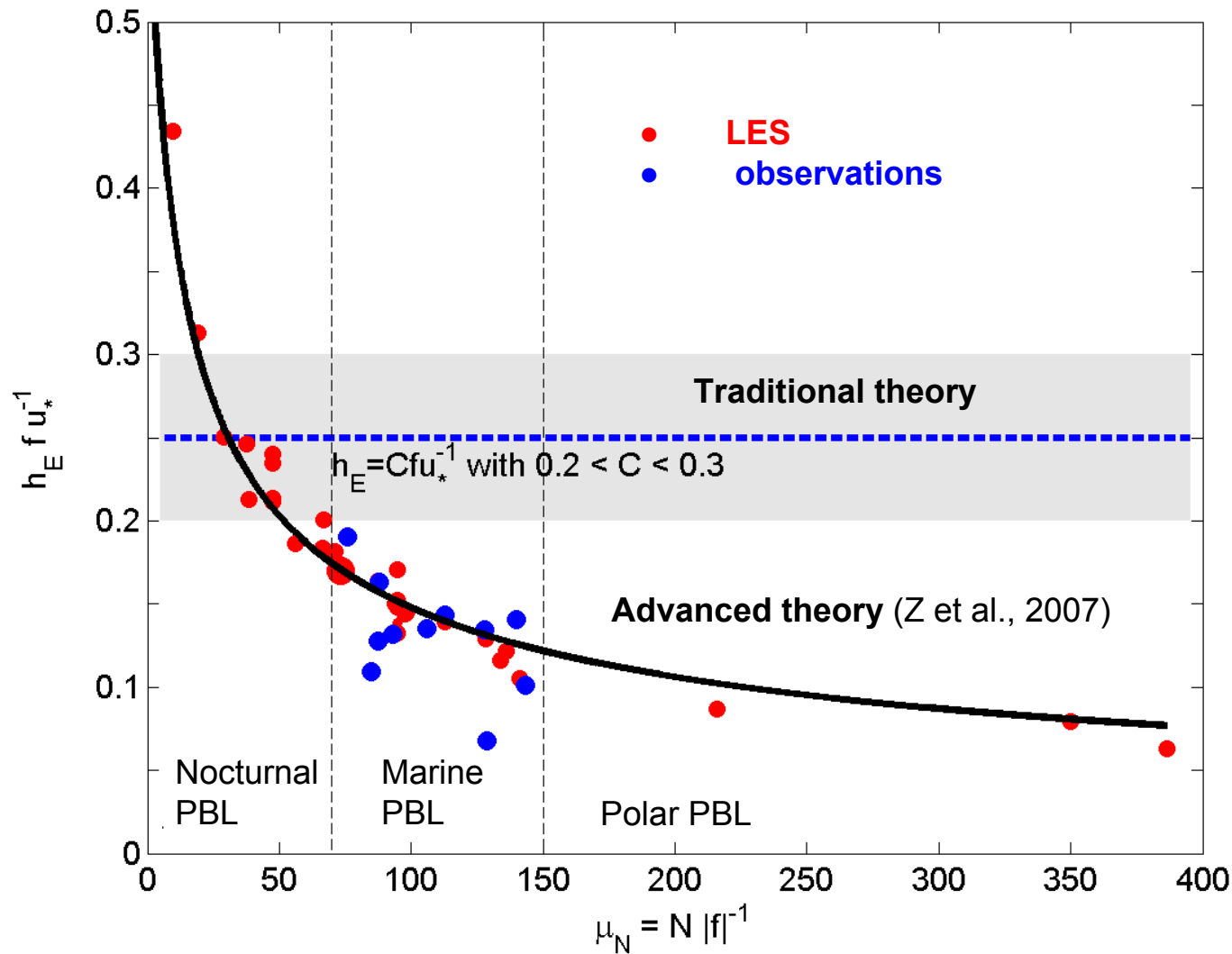
L I M I T S	{	$C_R u_*  f ^{-1}$	TN PBL Earth rotation ( $C_R = 0.6$ )
		$C_{CN} u_T  fN ^{-1/2}$	CN PBL free flow impact ( $C_{CN} = 1.4$ )
		$C_{NS} u_*^2  fB_s ^{-1/2}$	NS PBL surface-flux impact ( $C_{NS} = 0.5$ )

**Baroclinic deepening** (Z&E-2003)  $u_T = u_* (1 + C_0 S_g / N)^{1/2}$

$S = dU_g/dz$  is geostrophic shear,  $C_0 = 0.67$  (Z& Esau, 2003)

- CN and TN (R-M, 1935) were confused: strongly variable ratio  $|f|h_E / u_* = C_N (f/N)^{1/2}$  was treated as constant (see Figure)
- NS (Z, 1972) widely accepted (data: over land at mid latitudes)

# PBL shallowing due to free-flow stability



Dashed line – traditional TN PBL model

Heavy curve – CN PBL model

Red points – LES

Blue points – atmospheric data

The effect of free-flow Brunt-Väisälä frequency  $N$  on the equilibrium CN PBL height  $h_E$



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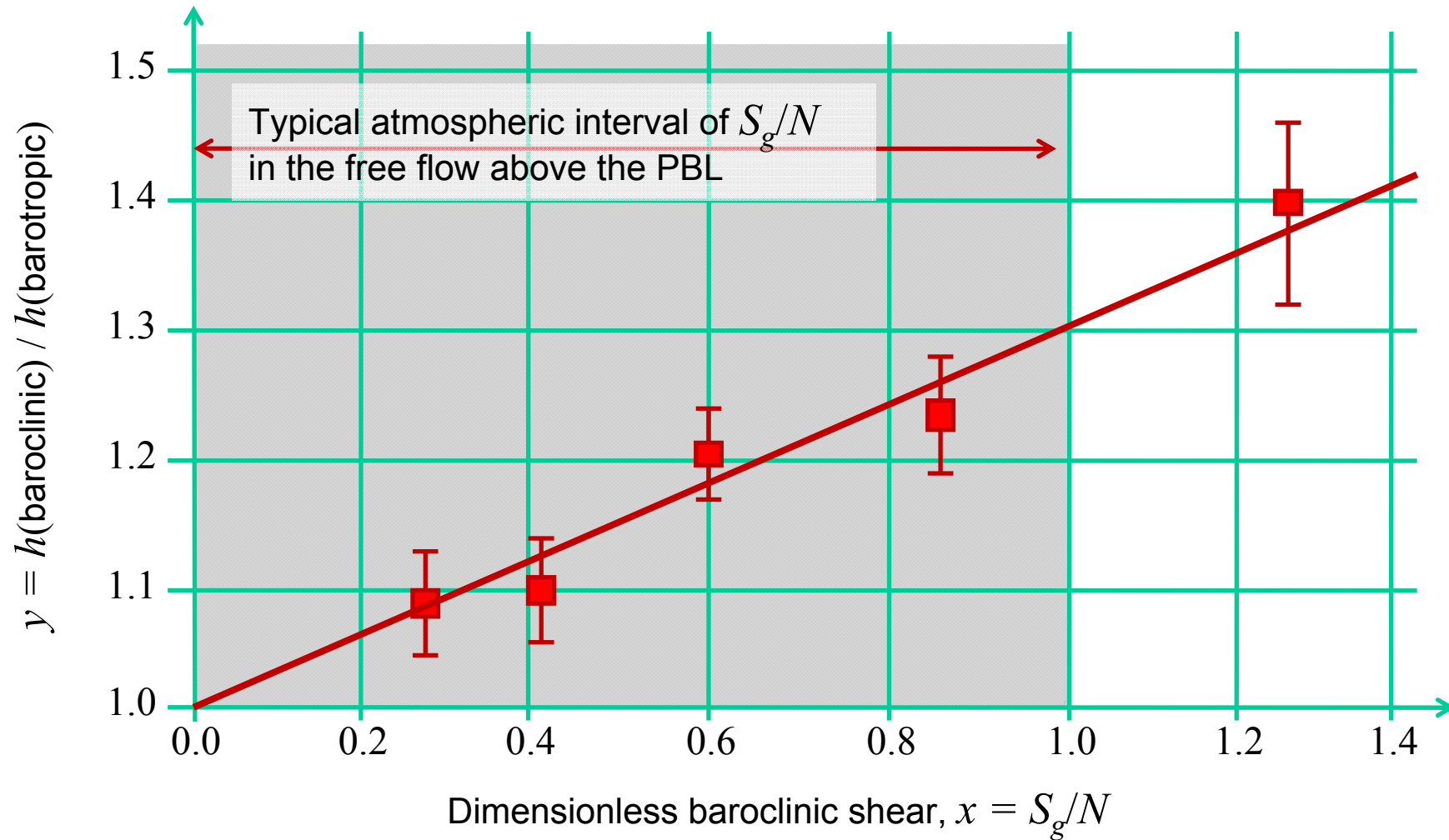


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# PBL deepening due to baroclinic shear

Theoretical model  $y = (1 + 0.67x)^{1/2}$  against LES (Zilitinkevich & Esau, 2003)





# Diagnostic and prognostic formulations

The equilibrium height  $h_E$  of stable or neutral PBL is controlled by rotation ( $f$ ) and stratification – at the bottom ( $B_s$ ) and on top ( $N$ )

$$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N|f|}{(C_{CN} u_*)^2} + \frac{|f B_s|}{(C_{NS} u_*^2)^2}$$

Given  $h_E$  real PBL height  $h$  is determined by relaxation equation

$$\frac{dh}{dt} - w = -\frac{h - h_E}{t_E}$$

Relaxation time scale  $t_E = \frac{h_E}{C_E u_*}$  includes empirical constant  $C_E$

Further work: extend to Equator  $\rightarrow$  ultimate CN PBL limit  $h_E \sim U/N$   
verify against LES atmospheric and oceanic data



# Long-lived stable PBL



Very shallow long-lived stable PBL visualised by smoke in warm summer day over cold Lake Teletskoe, Altay, Russia, 28.08.2010



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# “Long-lived” stable PBL



Rising smoke in Lochcarron (Scottish Highlands) blocked within shallow PBL capped by strong temperature inversion. Photo by S/V Moonrise (Wikimedia).



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# Convective boundary layers

Turbulent kinetic energy (TKE) budget equation

Kinetic energy is generated convectively and mechanically

Shear-free regime:

when **convective generation** is much stronger than **mechanical**

**Deardorff (1972)** governing parameters:

convective velocity scale:

$$W_* = (B_s h)^{1/3}$$

similarity theory:  $E_K / W_*^2 = \Phi_{EK}(z/h), \dots$

Eddy-viscosity scale  $K_* = W_* h$  substituted into Ekman height

scale  $h_* = (K_* / |f|)^{1/2}$  yields  $h_* \sim B_s^{1/2} / f^{3/2} \sim 30 \text{ km}$

much higher than atmospheric PBL height → **hence**

**ROTATION NEGLIGIBLE** → convective PBL ≠ Ekman layer



# Convective boundary layers

TKE-budget

$$\frac{dE_K}{dt} = \boldsymbol{\tau} \cdot \frac{\partial \mathbf{u}}{\partial z} + B - \frac{\partial F}{\partial z} - \varepsilon$$

Kinetic energy is generated convectively and mechanically with generation rates  $B > 0$  and  $\boldsymbol{\tau} \cdot \partial \mathbf{u} / \partial z$ , respectively

Shear-free regime: when convective generation  $\gg$  mechanical

**Deardorff (1972)** governing parameters:  
convective velocity scale:  $W_* = (B_s h)^{1/3}$   
similarity theory:  $E_K / W_*^2 = \Phi_{EK}(z/h), \dots$

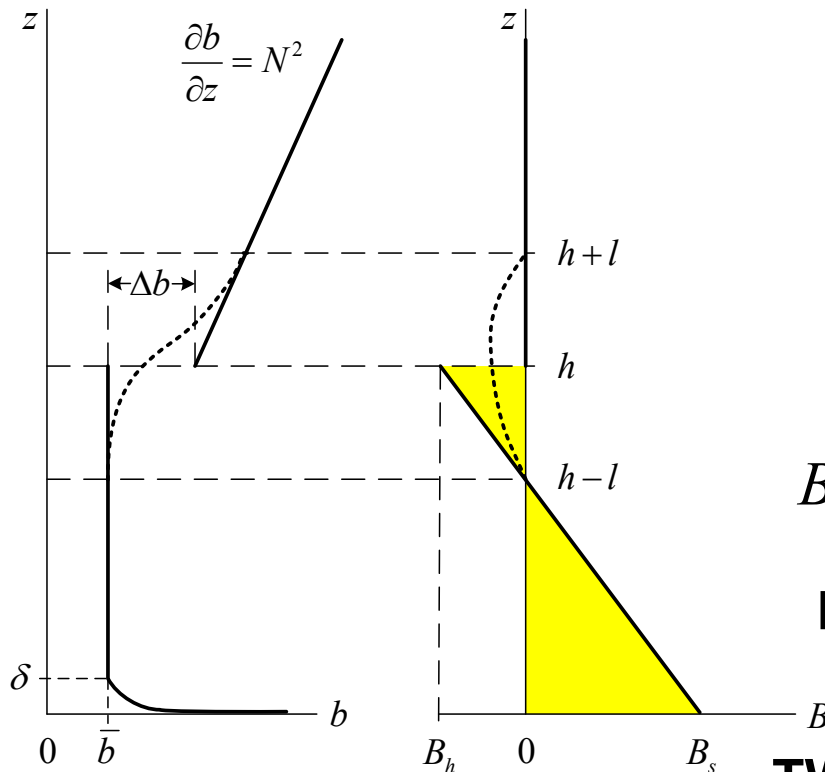
Eddy-viscosity scale  $K_* = W_* h$  substituted into Ekman height scale  $h_* = (K_* / |f|)^{1/2}$  yields  $h_* \sim B_s^{1/2} / f^{3/2} \sim 30$  km  
much higher than atmospheric PBL height  $\rightarrow$  **hence**

**ROTATION NEGLIGIBLE**  $\rightarrow$  convective PBL  $\neq$  Ekman layer



# The concept of well-mixed layer

Typically  $B_s \sim 10^{-3} \text{ m}^2\text{s}^{-3}$   $\rightarrow$  strong mixing  $\rightarrow$  buoyancy ( $b$ ), velocity ( $u, v$ ), etc. are  $z$ -independent, except near-surface ( $0 < z < \delta$ ) and entrainment ( $h - l < z < h + l$ ) layers  $\rightarrow$  almost linear vertical profiles of turbulent fluxes ( $B$  and  $\dot{h}$ )



$$b = \begin{cases} \bar{b} & \text{at } \delta < z < h \\ \bar{b} + \Delta b + N^2(z - h) & \text{at } z > h \end{cases}$$

$$B = \begin{cases} B_s(1 - z/h) + B_h z/h & \text{at } 0 < z < h \\ 0 & \text{at } z > h \end{cases}$$

$$B_h = -\Delta b \dot{h} < 0 \text{ caused by } \Delta b = b_{h+l} - b_{h-l}$$

Entrainment depth-scale ( $l$ )  
from similarity of triangles

$$\frac{l}{h-l} = -\frac{B_h}{B_s}$$

## TWO PROBLEMS

**1. Growth rate (PBL height)**

$$\dot{h} = \frac{dh}{dt} - w$$

**2. Entrainment coefficient (VENTILATION & CLOUDS)**

$$A = \frac{-B_h}{B_s} = \frac{\dot{h} \Delta b}{B_s}$$

# PBL-height buoyancy-budget models

Budget equation for mean buoyancy  $b = (g/T)\theta + 0.61gq$  is composed of the

potential temperature ( $\theta$ ) and specific humidity ( $q$ ) equations:  $\frac{db}{dt} = -\frac{\partial B}{\partial z}$

Taking the well-mixed-layer  $b(z)$  and  $B(z)$  and integrating over the PBL, yields bulk buoyancy equation with **TWO UNKNOWN**  $h$  and  $\Delta b$  ( $\partial B_h = -\Delta b \dot{h}$ ):

$$\frac{d}{dt} \left( \frac{1}{2} N^2 h^2 - h \Delta b \right) = B_s$$

No entrainment (Zubov, 1945;  $B_h = 0$ )  $\rightarrow$  encroachment model:

$$\dot{h} = \frac{B_s}{N^2 h}$$

Given entrainment (Betts/Carson/Tennekes, 1973:  $A = -B_h/B_s = \text{constant}$ )  $\rightarrow$

$$\dot{h} = (1 + 2A) \frac{B_s}{N^2 h}$$

**irrelevant** at  $N=0$  (in lab experiments  $\dot{h} \approx 0.24W_*$ )

used in Gryning-Batchvarova (1990,...) with  $A = 0.2$



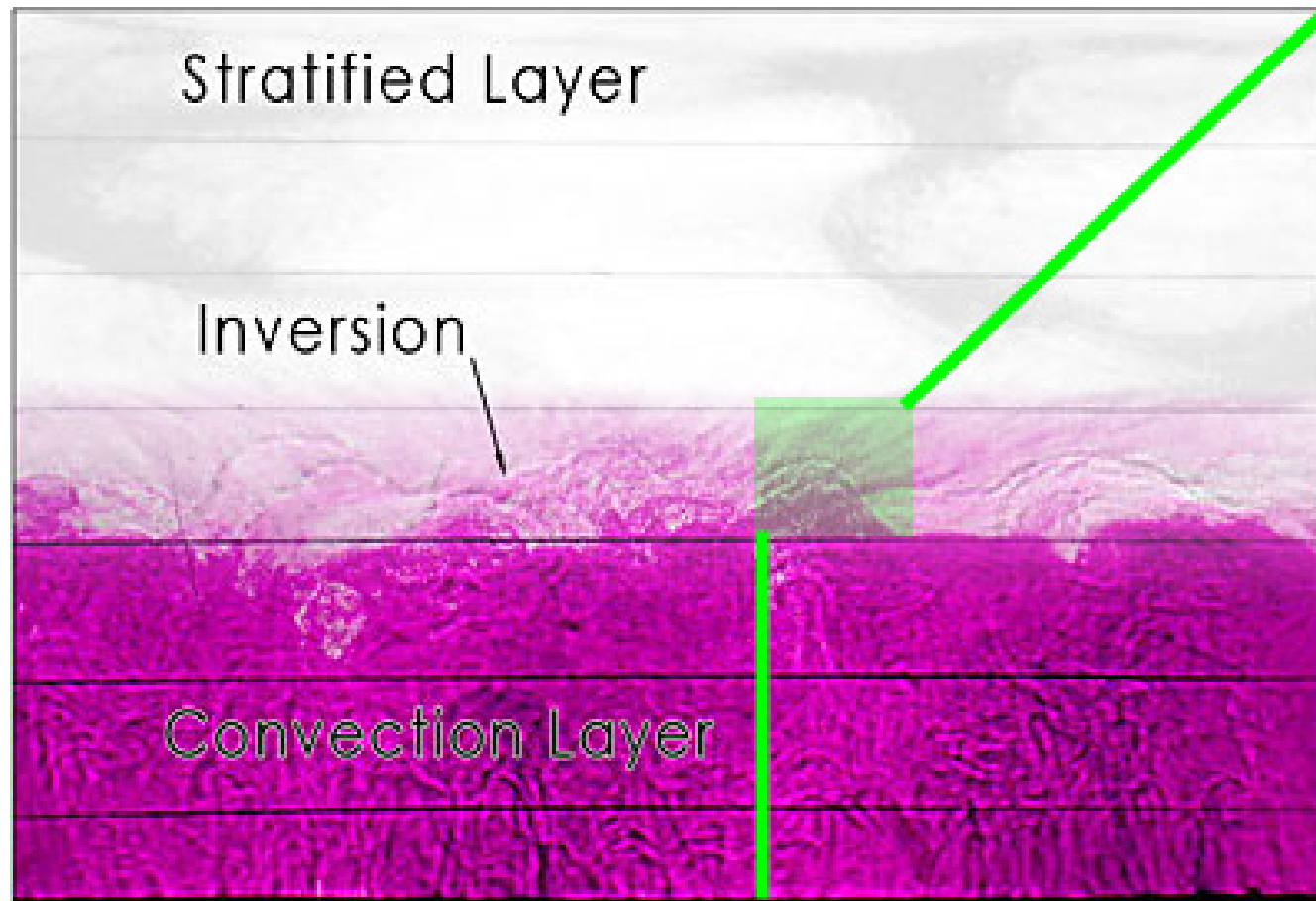


# Empirical data on $A = B_h/B_s$ (from Z, 1991)

Atmosphere	Laboratory	Lake	$A=B_h/B_s$
Koprov, Zwang 1965			0.35
Deardorff 1967			0.1 – 0.3
Lenschow , Johnson 1968			0.9
	Deardorff et al. 1969		0.12
Lenschow 1970			0.1
Deardorff 1972			0.1
Lenschow 1973			0 – 0.17
Stull 1976			< 0.1
Deardorff 1973			0.2
Carson 1973			0.25
Betts 1973			0.25
Cattle, Weston 1973			0.25
Lenschow 1974			< 0.12
Pennel, Le Mone			< 0.1
Rayment, Readings 1974			0.25
	Deardorff et al. 1974		0.23
Betts 1974			0.3
	Willis, Deardorff 1974		0.11 – 0.23
Cattle, Weston 1975			0.29 – 0.32
		Farmer 1975	0.2
	Heidt 1977		0.18
Yamamoto et al. 1077			0.2
	Kantha 1979		0.2
Coulman 1980			0.25
Dubosklard 1980			0.9
	Denton Wood 1981		0.2 – 0.5
Driedonks, Tennekes 1984			0.2



# Shear-free convection in laboratory



A snapshot of the convection layer in laboratory experiment  
Earth, Atmospheric & Planetary Sciences Project, MIT, 2008

[http://paoc.mit.edu/labguide/convect\\_atm.html](http://paoc.mit.edu/labguide/convect_atm.html)



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# Lab entrainment TKE-budget model (Z-91)

Two unknowns in bulk buoyancy-budget equation:

PBL height  $h$ , and the buoyancy increment due to entrainment  $\Delta b$  or entrainment coefficient

$$\frac{d}{dt} \left( \frac{1}{2} N^2 h^2 - h \Delta b \right) = B_s$$

$$A = \frac{\dot{h} \Delta b}{B_s}$$

Z (1975) closed the problem employing **bulk TKE-budget** equation and solved it (1987) for "laboratory" shear-free PBL growing against linear stratification accounting for energy consumption for exciting internal gravity waves at  $z > h$

$$A + C_3 \text{Ri}^{3/2} \left( \frac{A}{1+A} \right)^3 = C_1 - C_2 E$$

$E = \dot{h} / W_*$  and  $\text{Ri} = \frac{1}{2} (Nh / W_*)^2$  are Richardson number

$C_1 = 0.2$ ,  $C_2 = 0.8$ ,  $C_3 = 0.1$  are empirical constants (Z, 1991)

# Lab entrainment TKE-budget model (Z-91)

TKE-budget in shear-free convection (laboratory)

$$\frac{dE_K}{dt} = B - \frac{\partial F}{\partial z} - \varepsilon$$

Deardorff similarity:  $E_K = W_*^2 \Phi_{EK}(\zeta)$ ,  $\varepsilon = (W_*^3 / h) \Phi_\varepsilon(\zeta)$  where  $\zeta = z / h$

Well-mixed layer buoyancy-flux profile:

$$B = B_s(1 - \zeta) + B_h \zeta$$

Integration over PBL  $\rightarrow$  Internal-Gravity-Wave energy flux:

$$F_{h+0} \propto \lambda^2 \Lambda N^3$$

IGW length and amplitude:  $\Lambda \sim \lambda \sim l = \frac{A}{1+A} h$ ,  $l$  entrainment-layer depth

**SOLUTION**

$$\frac{d}{dt} \left[ W_*^2 \left( \text{Ri} - \frac{A}{E} \right) \right] = B_s \quad A + C_3 \text{Ri}^{3/2} \left( \frac{A}{1+A} \right)^3 = C_1 - C_2 E$$

Entrainment coefficient

$$A = \frac{\dot{h} \Delta b}{B_h}$$

Expansion rate

$$E = \dot{h} / W_*$$

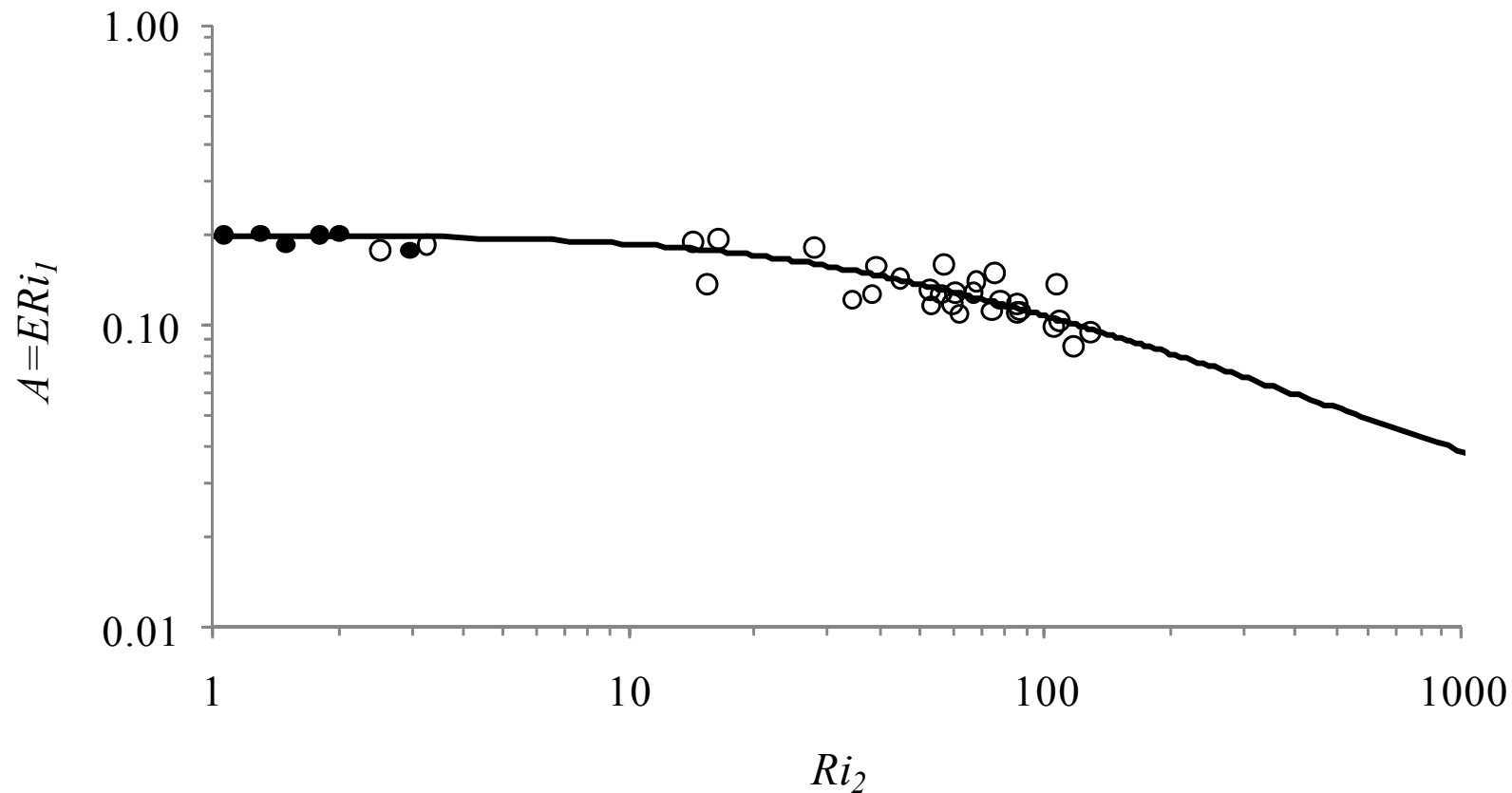
Ri number

$$\text{Ri} = \frac{1}{2} (Nh / W_*)^2$$

Empirical constants from lab experiments:  $C_1 = 0.2$ ,  $C_2 = 0.8$ ,  $C_3 = 0.1$



# Validation against lab-experiment data



Entrainment coefficient  $A = B_w/B_s$  versus  $Ri_2 = \frac{1}{2}(Nh/W_*)^2$  in Deardorff et al. (1980) laboratory experiments with shear-free convective layers evolving against linearly stratified fluid (black circles) and two-layer fluid (open circles). Theoretical curve shows TKE-budget entrainment model Z (1987, 1991) with  $C_1 = 0.2$ ,  $C_2 = 0.8$ ,  $C_3 = 0.1$ .



# Organised rolls in sheared convection



Self-organised rolls stretched along mean wind occupy the entire PBL ( $0 < z < h$ ) with distance between rolls of order  $2h$ .

Updraughts excite IGW with wave length typically  $\sim 2h$  in the stably stratified free atmosphere

## Cloud streets visualise updraughts in convective rolls

Photo by Mick Petroff, Queensland, North Coast, Australia (Wikimedia Commons)

# Atmospheric entrainment model (Z, 2011)

TKE-budget in sheared convection

$$\frac{dE_K}{dt} = \boldsymbol{\tau} \cdot \frac{\partial \mathbf{u}}{\partial z} + B - \frac{\partial F}{\partial z} - \varepsilon$$

Generalised similarity:  $W_*^3 = 2 \int_0^h B dz \approx h B_s$ ,

$$V_*^3 = \int_0^h (\boldsymbol{\tau} \cdot \partial \mathbf{u} / \partial z) dz \approx \bar{U} u_*^2$$

For TKE  $E_K = W_*^2 \Phi_{EK}^{(c)}(\zeta) + V_*^2 \Phi_{EK}^m(\zeta)$  and similarly for dissipation  $\varepsilon$

IGW energy flux:

$$F_{h+0} \propto \lambda^2 \Lambda N^3$$

IGW amplitude:

$$\lambda \sim l = \frac{A}{1+A} h$$

but wave-length (the above photo)  $\Lambda \sim h$

**Entrainment equation accounts for shear and IGW caused by rolls**

$$A + C_4 \text{Ri}^{3/2} \left( \frac{A}{1+A} \right)^2 = C_1 - C_2 E + C_5 \left( \frac{V_*}{W_*} \right)^3$$

$$A = \frac{\Delta b \dot{h}}{B_h}$$

$$E = \frac{\dot{h}}{W_*}$$

$$\text{Ri} = \frac{1}{2} \left( \frac{Nh}{W_*} \right)^2$$

$C_1 = 0.2, C_2 = 0.8$  lab exp.  
 $C_4, C_5$  to be determined



# Conclusions

**Stable / neutral layers** **Heights** from dozens to hundreds *m*

Weakly mixed, strongly affected by the free-flow stratification

**LS and CN layers typical of Polar, marine, coastal atmosphere**

**Shallow Polar and sub-Polar PBLs AMPLIFY GLOBAL WARMING**

**Shallow LS PBLs control heavy air-pollution episodes**

**Convective layers** **Heights** up to 3-5 *km*, **variable entrainment**

Organised cells/rolls  $W_* = (B_s h)^{1/3}$  **ENHANCE HEAT/MASS TRANSFER**

**Over open waters in Polar Ocean (especially polynias and leads)**

**In Tropics, e.g. over “Warm Pool Area”**

Strongly variable turbulent entrainment  $0 < A = -B_h / B_s < 1$

***A* controls cumulus clouds (Knight et al., 2007), which causes strong climate sensitivity to *A* (currently given constant = 0,2)**

**Challenge Operational modelling and monitoring PBL height**



# PBL height and air quality



Project “Information System for Management of Air Quality”, ENEA, Italy, 2008-2010



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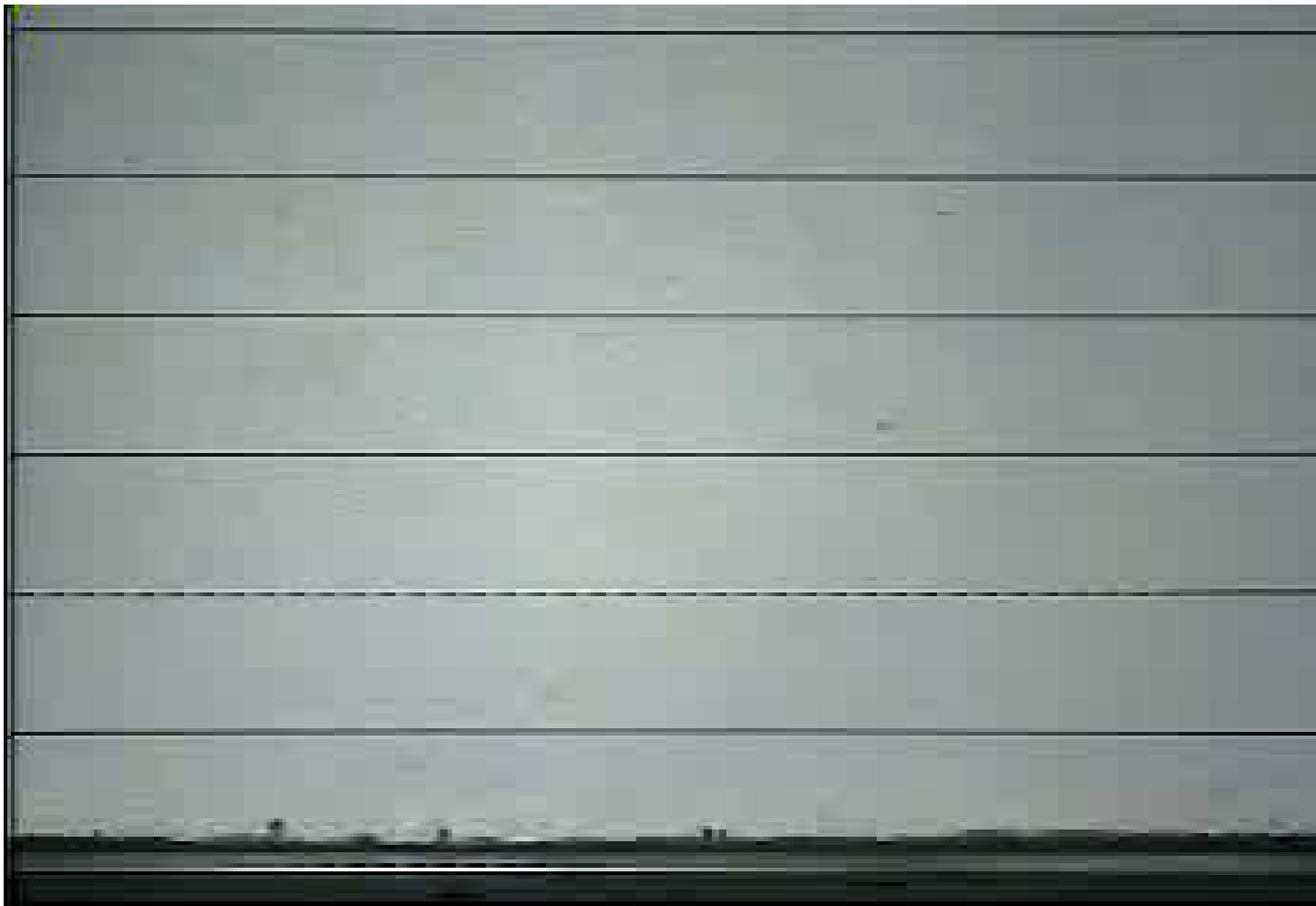


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# The evolution of convective boundary layer



Earth, Atmospheric and Planetary Sciences Project, MIT, 2008

[http://paoc.mit.edu/labguide/convect\\_atm.html](http://paoc.mit.edu/labguide/convect_atm.html)



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**Thank you  
for your attention**



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