Bending of annular plates

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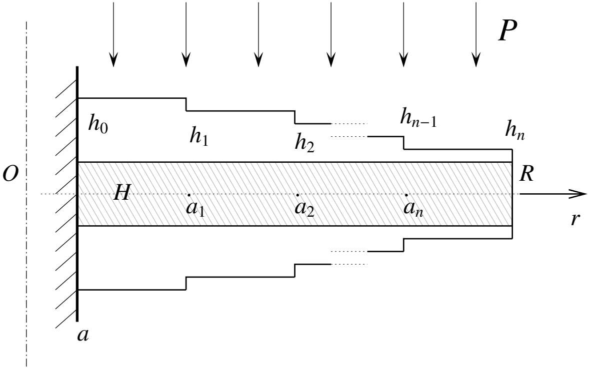
Outline

- Introduction
- Problem formulation
- Basic equations and concepts
- Numerical results
- Concluding remarks

Introduction

- Hodge, P. G. (1960)
- Chakrabarty, J. (2000)
- Kaliszky, S. and Logo, J. (2006)

Annular plate



Sandwich cross-section.

The aim is to determine the transverse deflection as well as the distributions of bending moments in the elastic and subsequent inelastic stages of deformation for given transverse pressure levels.

Basic equations and concepts

- Equilibrium conditions

$$\frac{d}{dr}(rM_1) - M_2 - rQ = 0, \quad \frac{d}{dr}(rQ) = -Pr$$

- Strain components

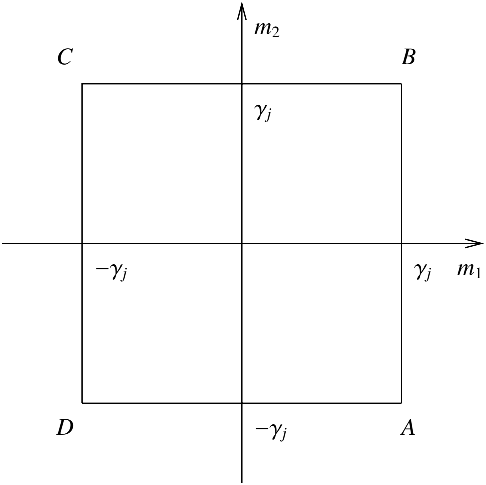
$$\kappa_1 = -\frac{d^2W}{dr^2}, \quad \kappa_2 = -\frac{1}{r}\frac{dW}{dr}$$

- Hooke's law

$$M_1 = D_j(\kappa_1 + \nu\kappa_2), \quad M_2 = D_j(\kappa_2 + \nu\kappa_1)$$

$$j = 0, \ldots, n \text{ and } D_j = \frac{Eh_jH^2}{2(1-\nu^2)}$$

Basic equations and concepts



Square yield condition.

Basic equations and concepts

- Plastic region

$$|M_1| \leq M_{0j}, \quad |M_2| \leq M_{0j}$$

$$M_{0j} = \sigma_0 h_j H$$

- Boundary conditions

$$M_1(R) = 0$$

$$Q(R) = 0$$

$$W(a) = 0$$

- Non-dimensional quantities

$$\rho = \frac{r}{R}, \quad m_1 = \frac{M_1}{M_*}, \quad m_2 = \frac{M_2}{M_*},$$

$$q = \frac{RQ}{M_*}, \quad \alpha = \frac{a}{R}, \quad \alpha_j = \frac{a_j}{R},$$

$$p = \frac{PR^2}{M_*}, \quad w = \frac{W}{H}, \quad \gamma_j = \frac{h_j}{h_*},$$

$$d_j = \frac{EH^2 h_j}{2(1-\nu^2)\sigma_0 R^2 h_*}$$

General solutions

- Elastic region

$$w = A_{1j}\rho^2 \ln\rho + A_{2j}\rho^2 + A_{3j}\ln\rho + A_{4j} + \frac{p\rho^4}{64d_j}$$

- Plastic region

$$w = A_j\rho + B_j$$

Stages

- Pure elastic stage of deformation (stage I)
- Elastic plastic stage with the hinge circle (stage II)
- Elastic plastic stage with a plastic region of finite length (stage III)

- Pure elastic stage of deformation (stage I)

Boundary conditions

$$w'(a) = w(a) = 0$$

$$m_1(1) = 0$$

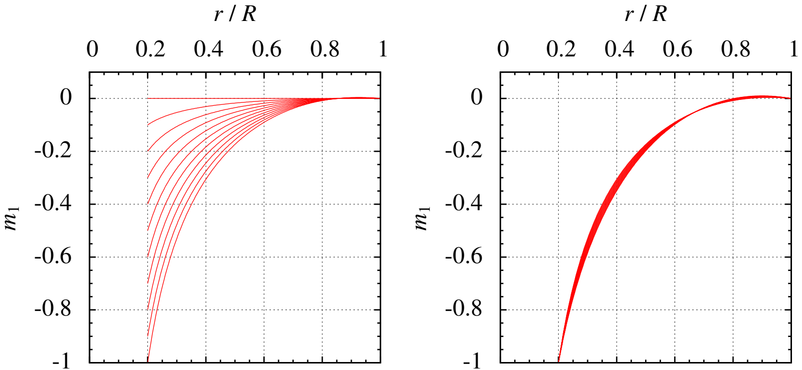
Continuity requirements

$$[w(\alpha_j)] = 0$$

$$[w'(\alpha_j)] = 0$$

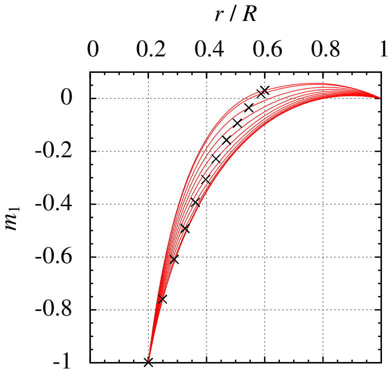
$$[m_1(\alpha_j)] = 0$$

Numerical results



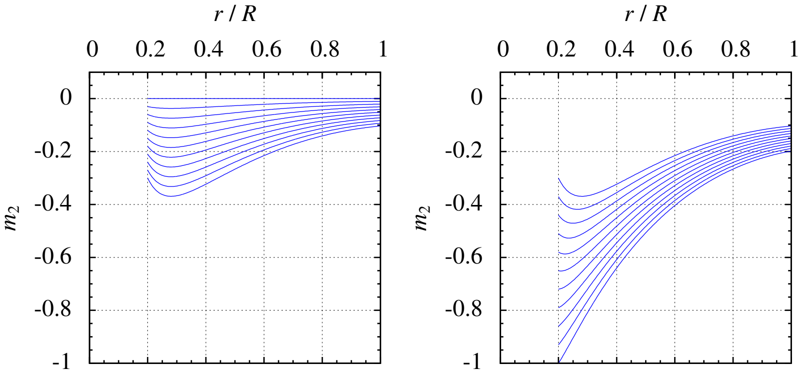
Bending moment m_1; stages I and II.

Numerical results



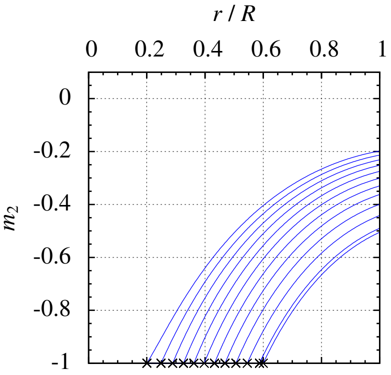
Bending moment m_1; stage III.

Numerical results



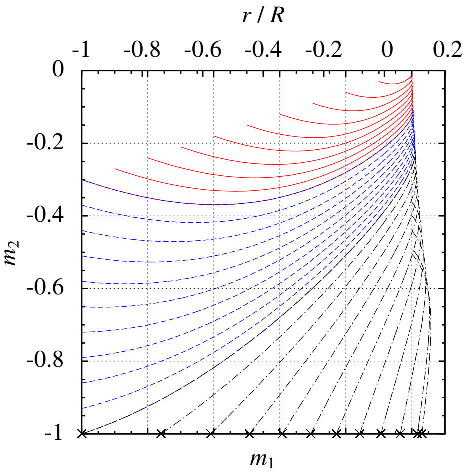
Bending moment m_2; stages I and II.

Numerical results



Bending moment m_2; stage III.

Numerical results



Bending moments m_1, m_2.

Concluding remarks

- A plate of variable thickness can be approximated with a plate of an appropriate choice of the piecewise constant thickness.
- The present technique is applicable for approximate solution of similar problems.

The end

Thank you for attention!