

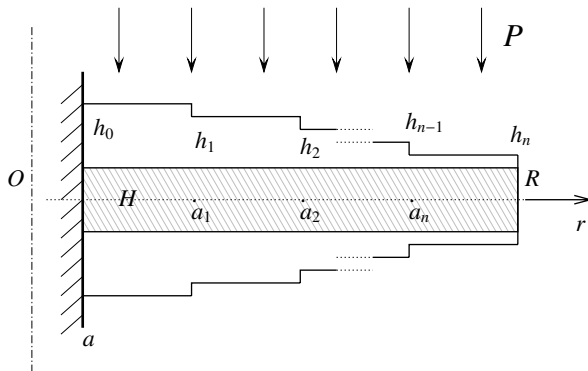
Elastic plastic bending of stepped annular plates

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Annular plate



Sandwich cross-section.

The aim of the paper is to determine the transverse deflection as well as the distributions of bending moments in the elastic and subsequent inelastic stages of deformation for given transverse pressure levels.

- Equilibrium conditions

$$\frac{d}{dr}(rM_1) - M_2 - rQ = 0, \quad \frac{d}{dr}(rQ) = -Pr$$

- Strain components

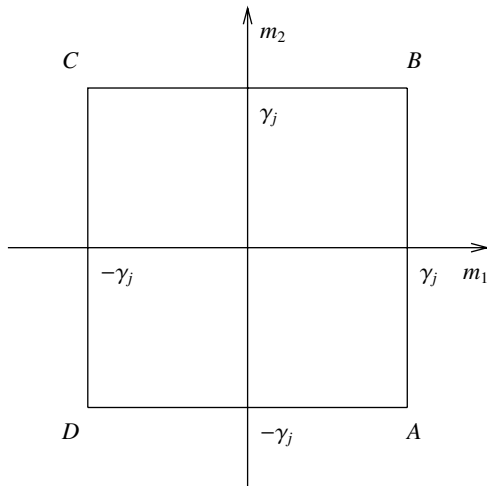
$$\kappa_1 = -\frac{d^2 W}{dr^2}, \quad \kappa_2 = -\frac{1}{r} \frac{dW}{dr}$$

- Hooke's law

$$M_1 = D_j(\kappa_1 + \nu\kappa_2), \quad M_2 = D_j(\kappa_2 + \nu\kappa_1)$$

$$j = 0, \dots, n \text{ and } D_j = \frac{Eh_j H^2}{2(1 - \nu^2)}$$

Basic equations and concepts



Square yield condition.

- Plastic region

$$|M_1| \leq M_{0j}, \quad |M_2| \leq M_{0j}$$

$$M_{0j} = \sigma_0 h_j H$$

- Boundary conditions

$$M_1(R) = 0$$

$$Q(R) = 0$$

$$W(a) = 0$$

- Non-dimensional quantities

$$\rho = \frac{r}{R}, \quad m_1 = \frac{M_1}{M_*}, \quad m_2 = \frac{M_2}{M_*},$$

$$q = \frac{RQ}{M_*}, \quad \alpha = \frac{a}{R}, \quad \alpha_j = \frac{a_j}{R},$$

$$p = \frac{PR^2}{M_*}, \quad w = \frac{W}{H}, \quad \gamma_j = \frac{h_j}{h_*},$$

$$d_j = \frac{EH^2 h_j}{2(1 - \nu^2)\sigma_0 R^2 h_*}$$

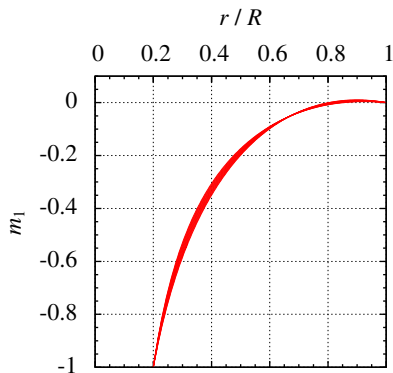
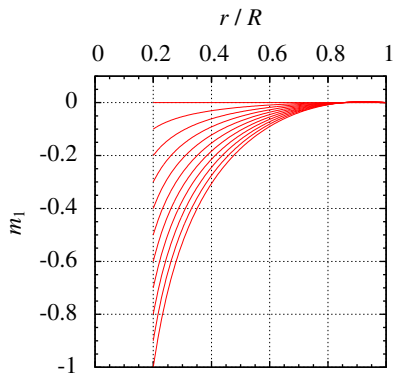
- Elastic region

$$w = A_{1j}\rho^2 \ln \rho + A_{2j}\rho^2 + A_{3j} \ln \rho + A_{4j} + \frac{p\rho^4}{64d_j}$$

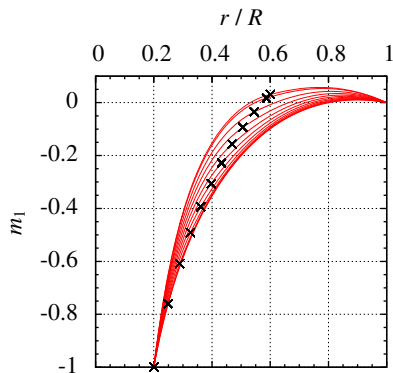
- Plastic region

$$w = A_j\rho + B_j$$

Numerical results

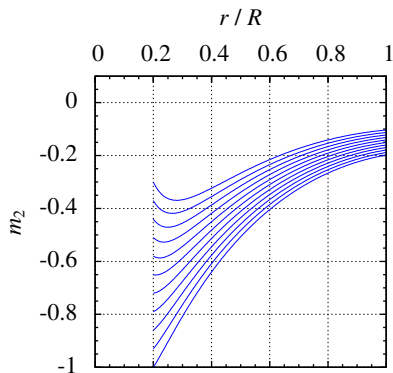
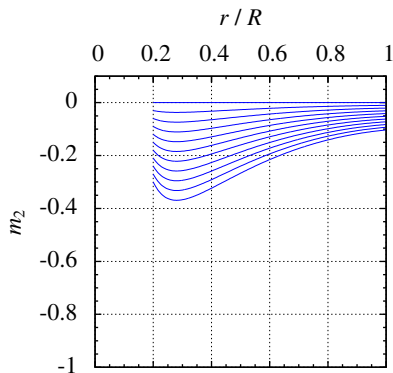


Bending moment m_1 ; stages I and II.



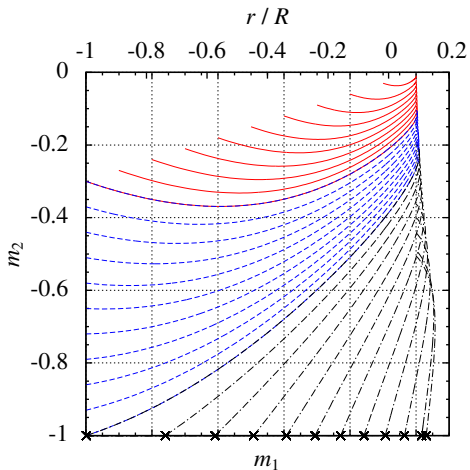
Bending moment m_1 ; stage III.

Numerical results



Bending moment m_2 ; stages I and II.

Numerical results



Bending moments m_1, m_2 .

